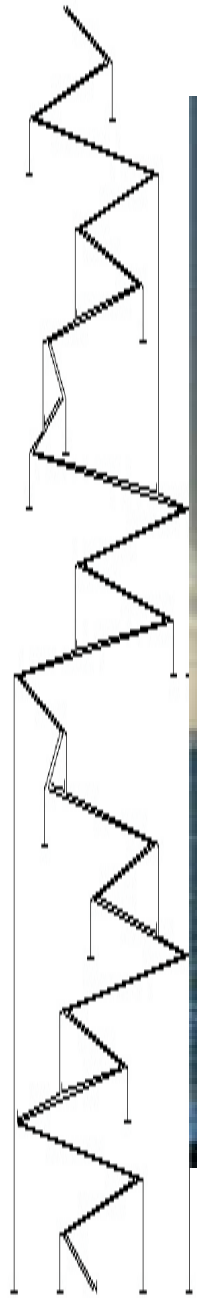
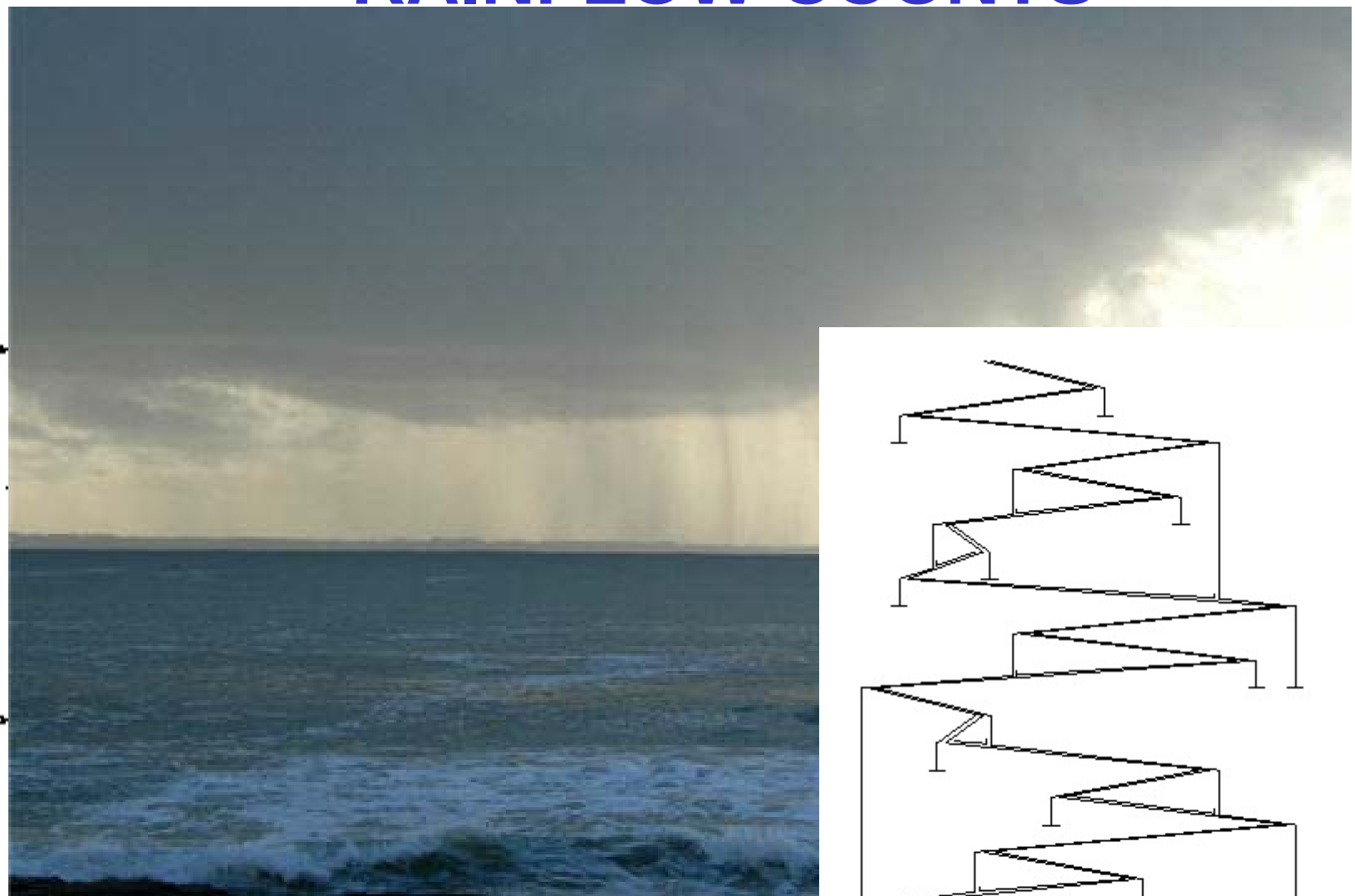
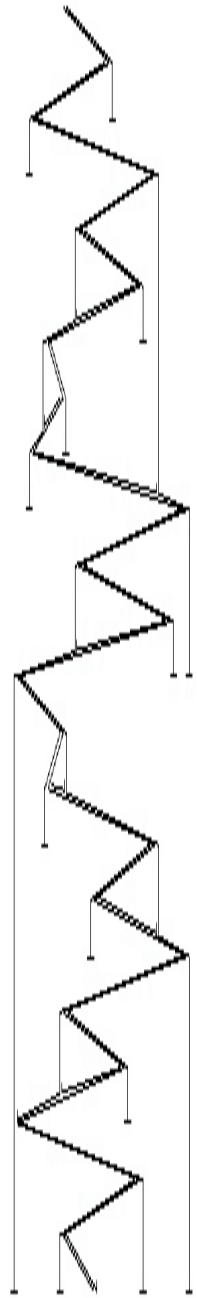
RAINFLOW COUNTS



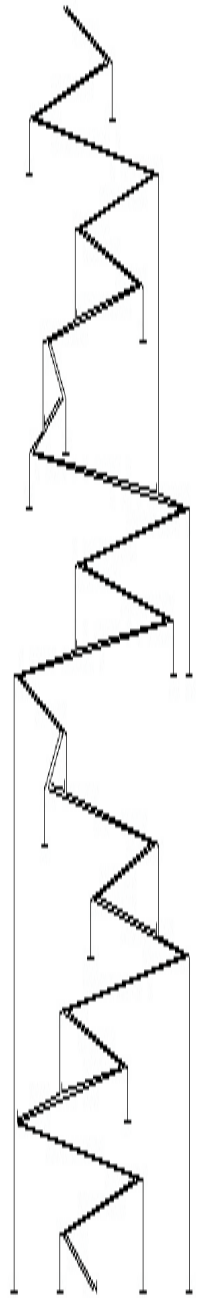


From fatigue to finance ?



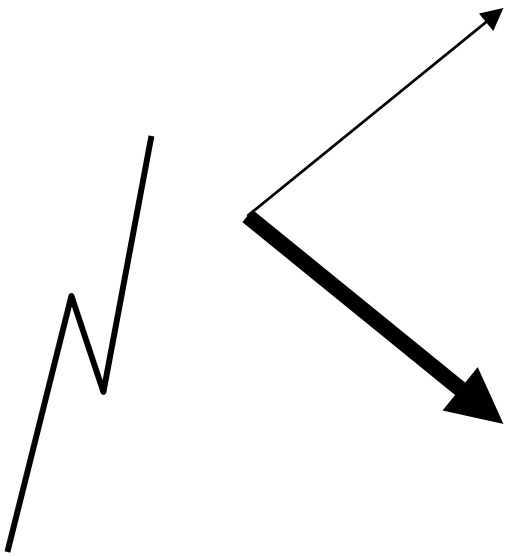
RAINFLOW COUNTS

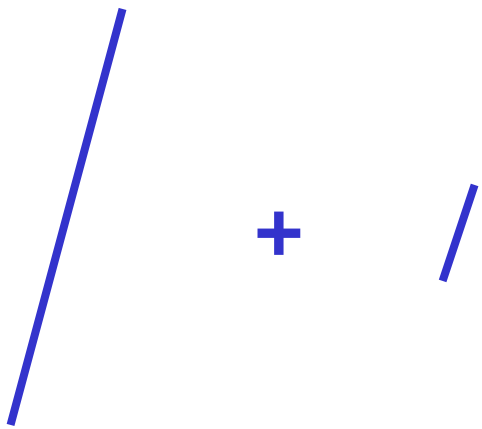
Rainflow is a nice way to separate small, uninteresting oscillations from the large ones, without affecting turning points by the smoothing effect of a filter nor interrupting a large range before it is actually completed.
Especially, in fatigue damage calculations, small amplitude ranges can often be neglected because they do not cause the cracks to grow.
Finance may have similar concerns, since the smallest amplitude ranges cannot be as profitable as larger ones because of broker costs, and since the trader is often trying to ignore them and to trade longer ranges.



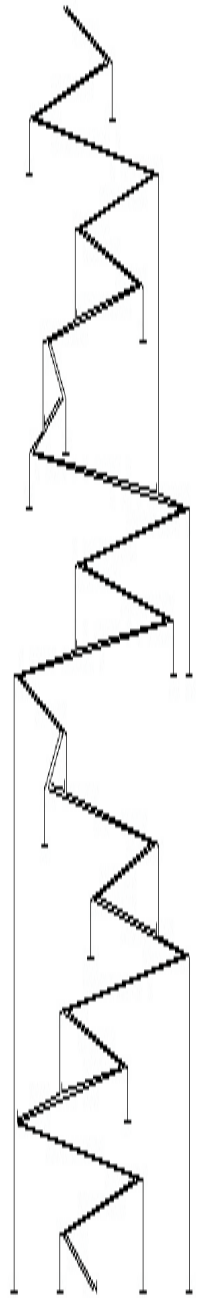
RAINFLOW COUNTS

Purpose of rainflow counting: consider a range with a wiggle on the way, it can be split into two half-cycles in several ways.





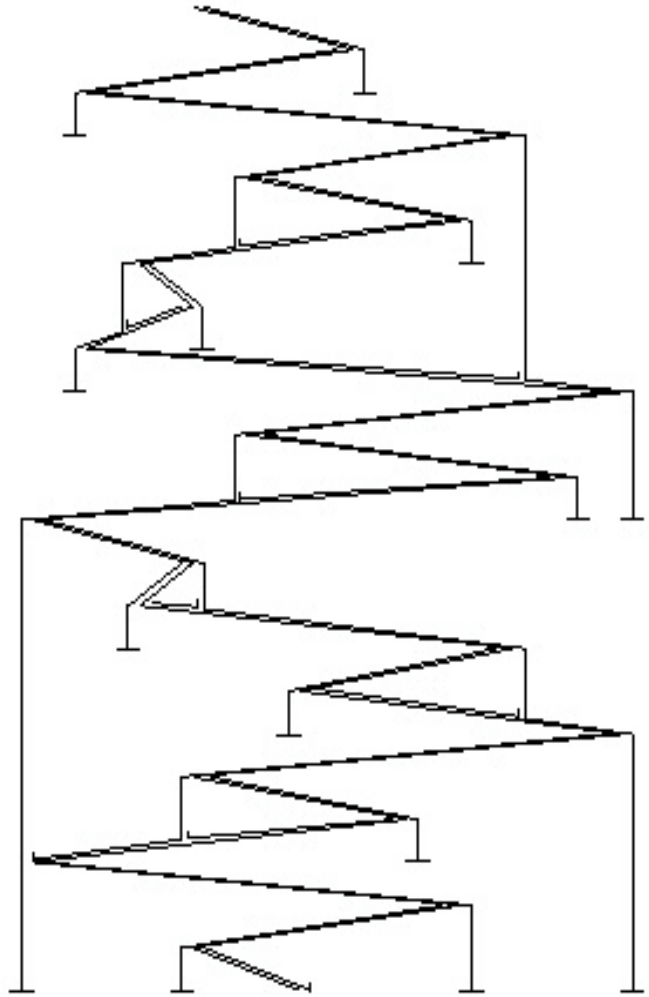
MUCH BETTER !



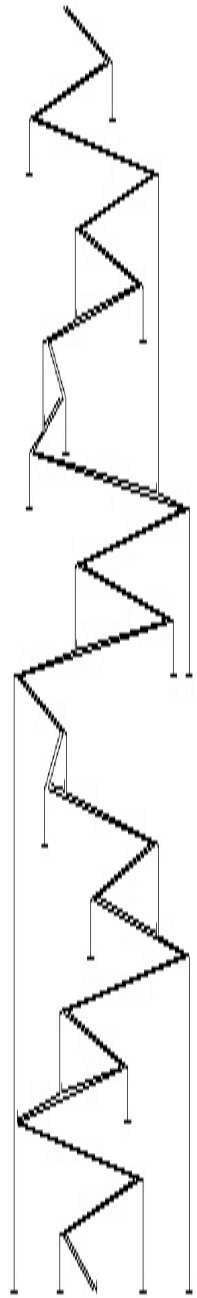
RAINFLOW COUNTS

Rainflow counting:
Let not small oscillations (small cycles) stop the “flow” of large amplitude ones.

Rainflow was originally defined by T. Endo as an algorithm (1974), another equivalent algorithm (de Jonge, 1980), that is easier to implement, is generally recommended (ASTM,1985), and a pure mathematical definition was first found by I. Rychlik (1987).



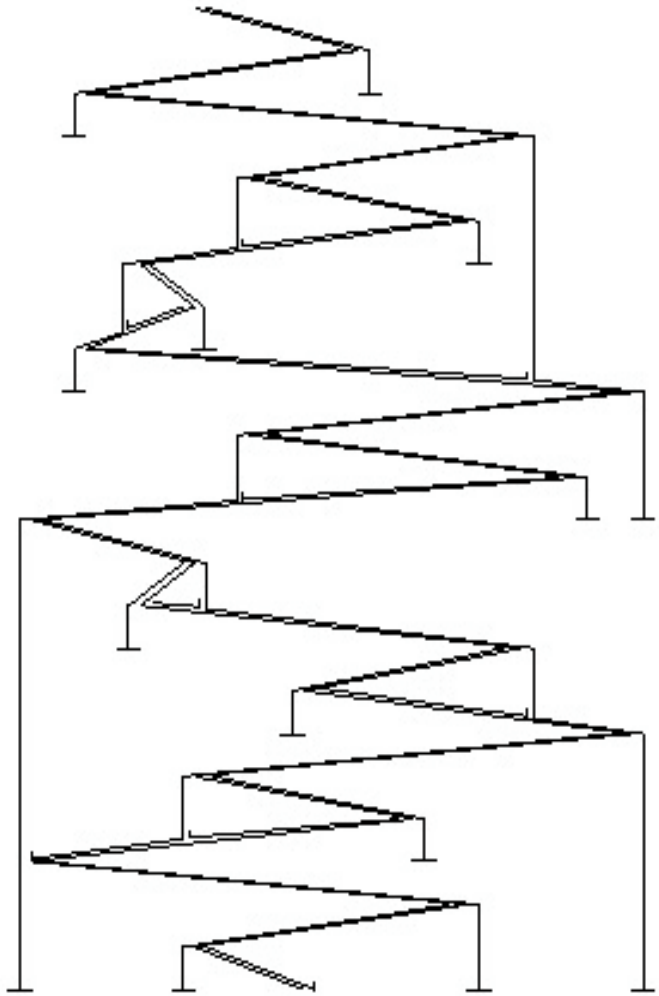
RAINFLOW COUNTS

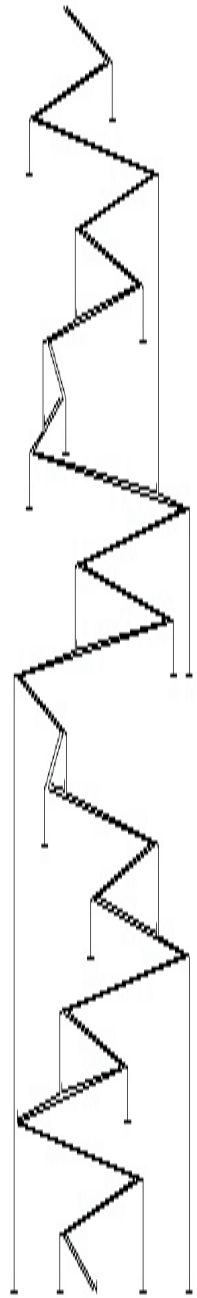


The Endo algorithm:

- **turn the signal around by 90°**
- **make water flow from their upper tops on each of the "pagoda roofs" so defined, until either a roof extends opposite beyond the vertical of the starting point, or the flow reaches a point that is already wet.**

Each time, a half-cycle is so defined, and most of them can be paired into full cycles (all could be if the signal was infinite).





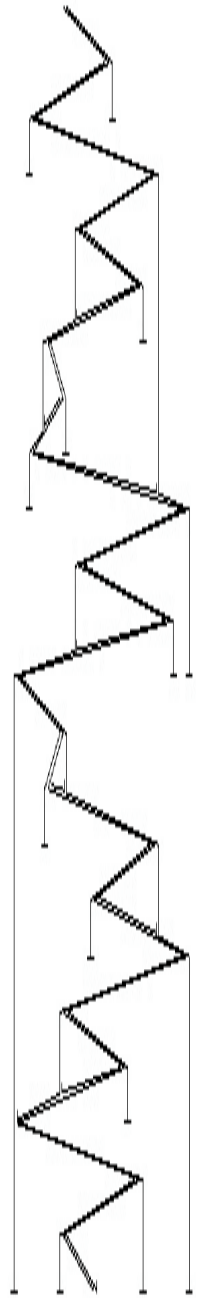
RAINFLOW COUNTS

The ASTM algorithm: for any set of 4 consecutive turning points,

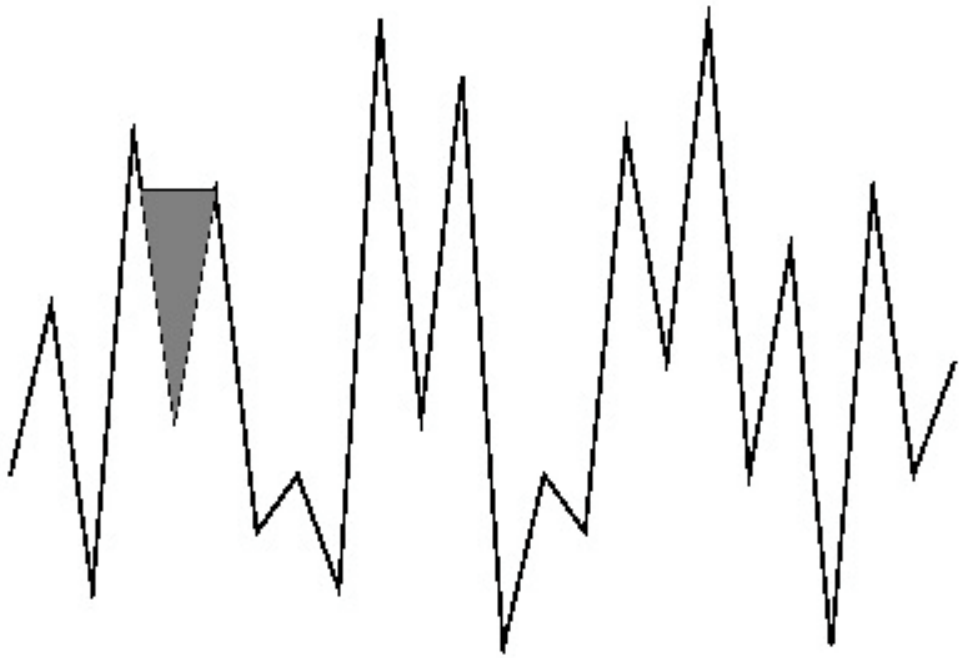
•Compute the 3 corresponding ranges (absolute values)

• If the middle range is smaller than the two other ones, extract a cycle of that range from the signal and proceed with the new signal

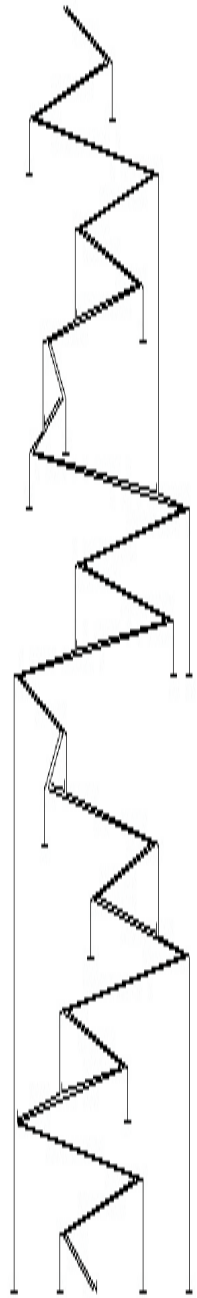
The remaining ranges when no more cycle can be extracted give a few additional half cycles (the same as the unpaired ones in the Endo algorithm).



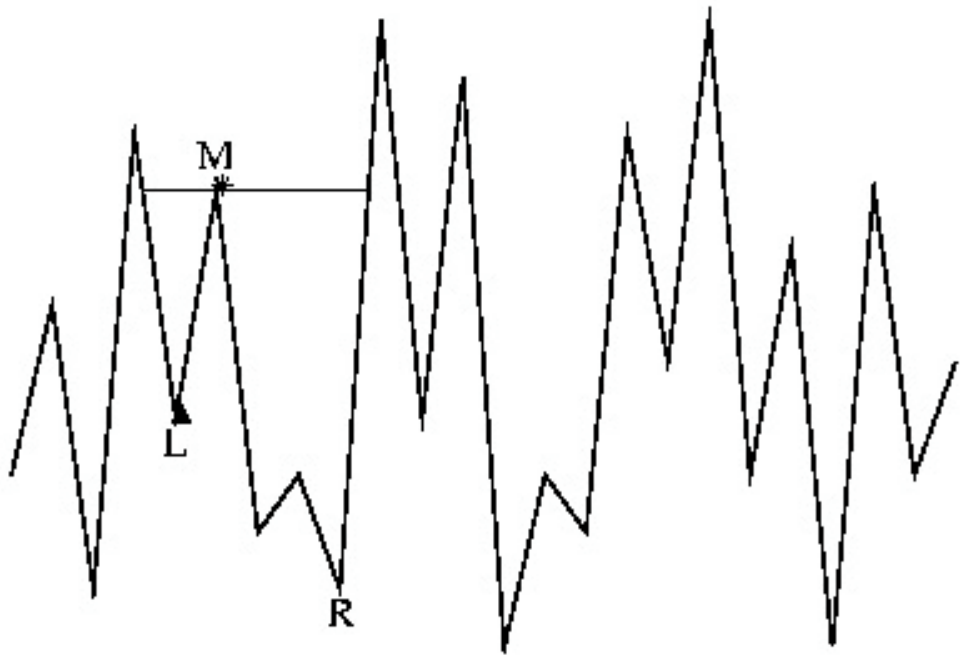
RAINFLOW COUNTS



The same ASTM algorithm is sometimes also called “rainfill”.

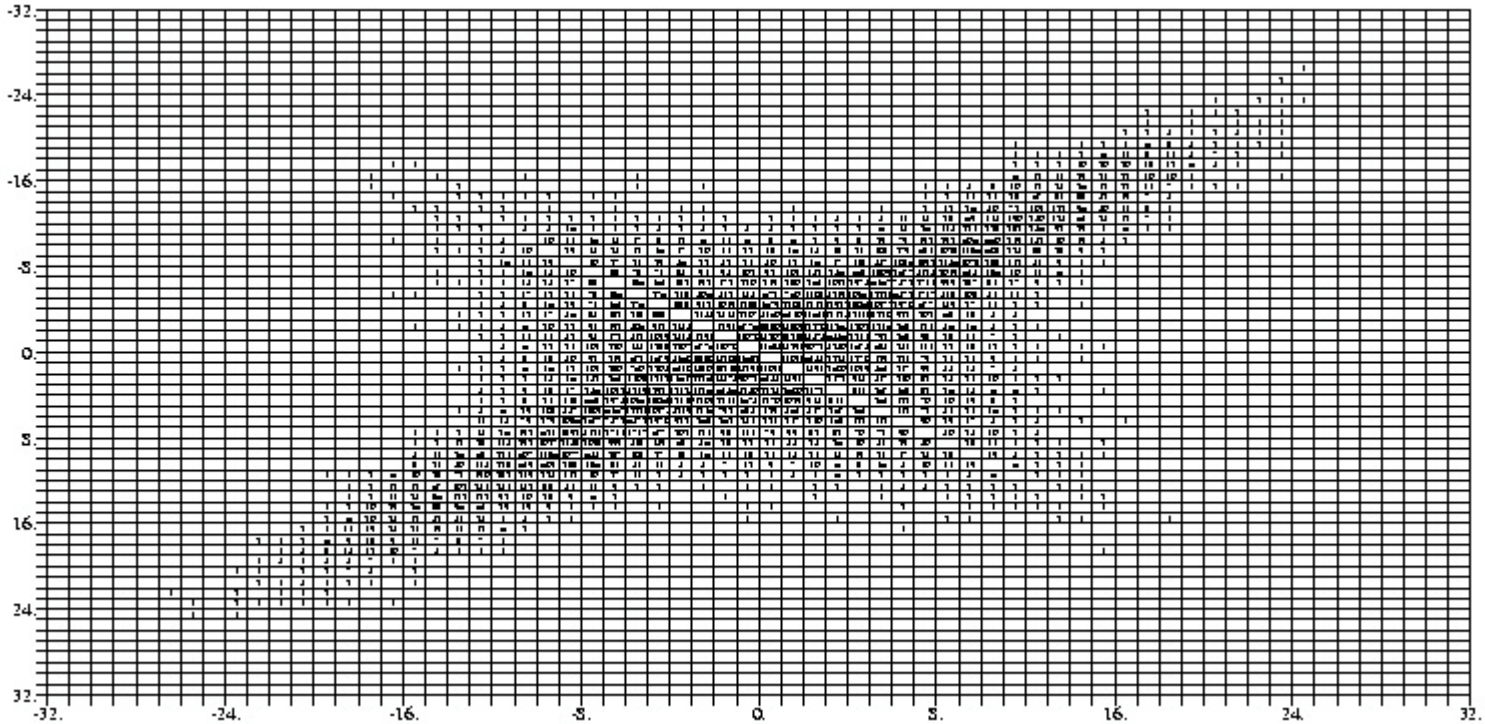


RAINFLOW COUNTS

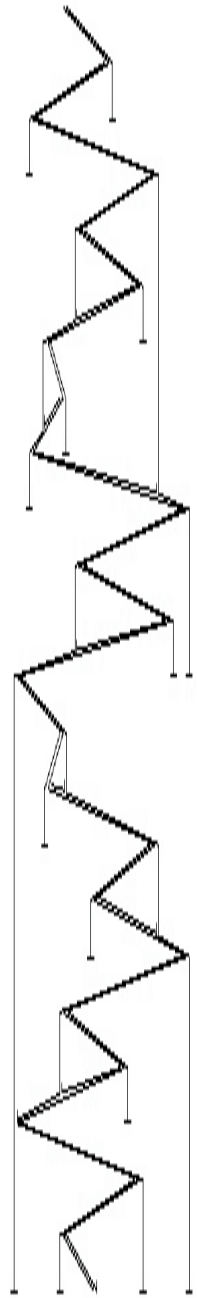


The mathematical definition: ***Consider a local maximum M. The corresponding minimum in the extracted rainflow count of cycles is max(L, R) where L and R are the overall minima on the intervals to the left and to the right until the signal reaches once again the level of M.***

RAINFLOW COUNTS



The mathematical definition allows, for discrete levels, to calculate the rainflow transition probability matrix from the min-max transition matrix of a Markov process. Thus, for instance, if the min-max only were recorded on a past experiment, a rainflow count can still be computed.

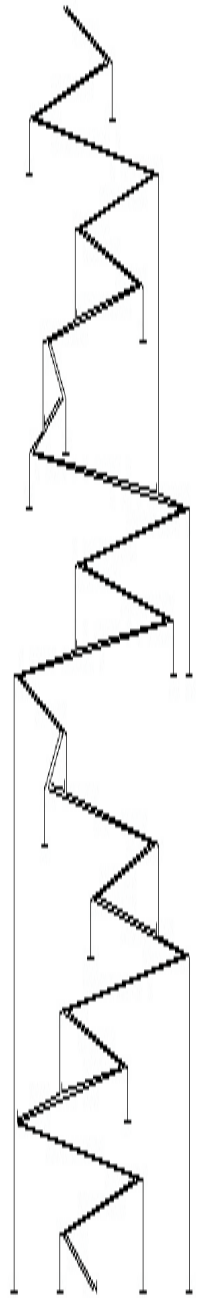


RAINFLOW COUNTS

When dealing with stationary Gaussian processes, a number of theoretical results are available that enable in most cases to compute the rainflow count (and the corresponding fatigue damage) exactly though not always quickly.

The turning points have distribution $\sigma(\sqrt{1-\varepsilon^2}R+\varepsilon N)$ where N is a normalized normal distribution, R a normalized Rayleigh distribution, σ is the standard deviation of the signal and ε is the spectral bandwidth parameter.

When the narrow-band approximation can be used ($\varepsilon=0$), and damage is of the Miner form ($D=\Sigma\nu_i\rho_i^m$, where ν_i is the number of cycles of range ρ_i), damage can be computed in closed-form since the moments of the Rayleigh distribution are related to the Γ function and ranges can be taken as twice the amplitude of turning points.



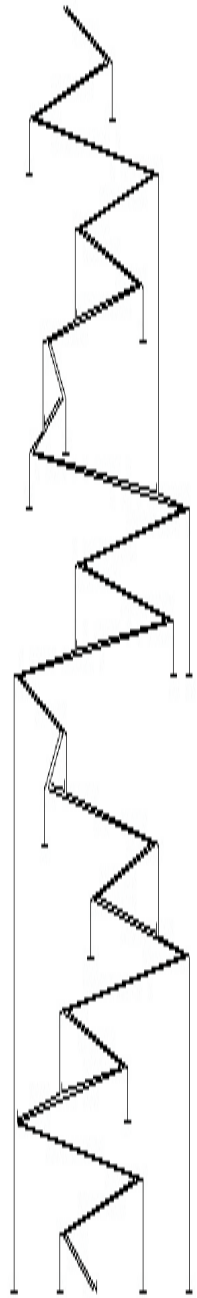
RAINFLOW COUNTS

When the narrow-band approximation cannot be used, the following results apply:

• The narrow-band approximation provides conservative estimates of the actual rainflow damage.

• If a power spectral density is given for the signal, the rainflow transition probabilities and range densities can be computed exactly, and thus also the fatigue damage. The calculations are complex, but readily implemented in the WAFO Matlab toolbox.

• If the original process is not gaussian, the use of an appropriate transformation to bring its probability density to the normal distribution is generally sufficient to have the gaussian process results apply to the transformed process (though “gaussian process” implies more than just normal probability density for the signal).



RAINFLOW COUNTS

Of course, in fatigue studies aim to minimize the potential damage and to find conservative estimates for it. In finance, the aim should be to maximize the damage that one causes to the market, i.e. the profit that one retrieves from it. Yet, the use of rainflow counting or rainflow filtering (ignore those rainflow cycles below some threshold) should provide both insight into past markets histories and statistics useful for comparison of the present to the past.